## Notes - Momentum

## Momentum and Impulse

- The product (multiplication) of an objects mass and velocity is called momentum. Momentum is the energy of motion of an object. Momentum is represented by the letter $p$.
- Earlier we learned that force equals mass multiplied by acceleration. Since mass is involved in the calculation of force we understand that more massive objects will take more force to stop.
- The equation to calculate the momentum of an object is $\vec{p}=m \vec{v}$. The units of momentum are $\times \frac{m}{s}$.
- An impulse is a force being applied to an object for an amount of time. An impulse is a vector as it is a force (magnitude and direction) in the direction of the force. The units must be $N \times s$ as an impulse is force multiplied by time.
- If an impulse is force exerted for a time interval then $m \Delta \vec{v}=\vec{F} \Delta t$
and we know $\Delta \vec{p}=m \Delta \vec{v} \quad$ so $\quad \Delta \vec{p}=\vec{F} \Delta t$.
- Ex. - A baseball of mass 0.14 kg is moving $a t+35 \frac{\mathrm{~m}}{\mathrm{~s}}$. What is the momentum of the baseball?

Answer -

- Ex. - Find the velocity at which a bowling ball of mass 7.26 kg would have the same momentum as the baseball.

Answer -

- Ex. - A 0.144 kg baseball is pitched horizontally at $+38 \frac{\mathrm{~m}}{\mathrm{~s}}$. After it is struck by the bat, it moves horizontally at $-38 \frac{\mathrm{~m}}{\mathrm{~s}}$. What impulse did the bat deliver to the ball?

Answer -

- Ex. - If the bat and ball were in contact for 0.80 ms , what was the average force exerted on the ball?
- Ex. - What was the average acceleration of the ball during contact with the bat?


## Answer -

## Conservation of Momentum

- When an object applies a force to another object there are three possible results of the impulse.
1.) The force applied causes an elastic result. An elastic collision is when contact is made between two objects and they bounce off each other and continue separately.
2.) The force applied causes an inelastic result. An inelastic collision is when contact is made between two objects and they stick together and continue off together.
3.) The force applied causes a semi-elastic result. A semi-elastic collision is when contact is made between the two objects and they stick together briefly before separating from each other and continuing separately.
- We know from the forces chapter that when a force is exerted on an object the object also exerts a force back. That is if you hit a baseball the bat exerts a force on the baseball, but likewise, the ball exerts a force back on the bat as well. The impulse from the ball must be in the opposite direction as the bat and subsequently the momentum of the bat must change.
- To study these forces and the results we need to look at the objects as a system. A system is a collection of specified objects that we are studying. The system is said to be a closed-system if objects neither enter or leave and open if an object enters or leaves.
- The system can also said to be isolated if no net external force is exerted on it. Two billiard balls on a pool table are a system and as long as friction is low, and if neither ball hits a bumper it is also isolated.
- We will continue to look at the example of billiard balls to look at the forces occurring in a collision.
- Diagram -

- In the above diagram ball $A$ is moving with momentum $\vec{p}_{A}$ and ball $B$ is moving with a greater momentum of $\vec{p}_{B}$. Ball $B$ will strike ball $A$ and exert an impulse of $+\vec{F} \Delta t$. The momentum of ball $A$ is now changed by an amount equal to the impulse as $\Delta \vec{p}=\vec{F} \Delta t$. The new momentum of $A$ (or $\vec{p}_{A}^{\prime}$ ) is $\vec{p}_{A}^{\prime}=\vec{p}_{A}+\Delta \vec{p}$.
- During the above collision, ball $A$ exerts a force back on ball $B$ that is equal to the impulse that changed its momentum but in the opposite direction. So ball B received an impulse of $-\vec{F} \Delta t$. This changed ball B's momentum by the amount of the impulse or $-\Delta \vec{p}$. Ball $B$ now has a momentum of $\vec{p}_{B}^{\prime}=\vec{p}_{B}+(-\Delta \vec{p})$.
- All of these changes show us that the momentum of ball $A$ increases and ball $B$ decreases. The part to note is that the amount of momentum ball A gains is the amount of energy ball B losses. The energy for the system has a net change of zero. Just like in chemical reactions the energy you start with is the energy you finish with.
- The law of conservation of momentum states that the momentum in any closed, isolated system does not change.
- Ex. - Two train cars with a mass of $3.0 \times 10^{5} \mathrm{~kg}$ each are sitting on a frictionless train track. This is a closed, isolated system. Car B is moving at $+2.2 \frac{\mathrm{~m}}{\mathrm{~s}}$ and car A is stopped. If the cars collide and move off together, find the momentum of each car after the collision and the momentum of the system before and after.

Answer -

- Ex. - A disk from an air hockey table of mass 0.355 kg moves along a frictionless air track with a velocity of $+0.095 \frac{\mathrm{~m}}{\mathrm{~s}}$. It collides with an air hockey stick of mass 0.710 kg moving in the same direction at a speed of $+0.045 \frac{\mathrm{~m}}{\mathrm{~s}}$. After the collision, the puck continues in the same direction with a velocity of $+0.035 \frac{\mathrm{~m}}{\mathrm{~s}}$. What is the velocity of the hockey stick after the collision?


## Answer -

- It is very important to define what the system is. Forces that occur within that system are called internal forces. The train cars colliding is an example of internal because we defined both train cars as being part of the system. If however the system was defined as just train car $A$ then when train car $B$ hits train car $A$ an external force has acted because car $B$ was outside of the system.
- Let's try another problem. This problem is to highlight that each force has an equal and opposite force resulting.
- Ex. - An astronaut at rest in space with mass of 84 kg fires a thruster that expels 35 g of hot gas at $+875 \frac{\mathrm{~m}}{\mathrm{~s}}$. What is the velocity of the astronaut after firing the shot?

Answer -

## Conservation of Momentum in Two Dimensions

- Two dimensional collisions are when objects collide and don't go off in the same or opposite directions. Often the objects will go off at angles to the initial velocity. These are two dimensional collisions.
- Two dimensional collisions are just like the chapter on vectors. The objects are heading off with both $X$ and Y components (at an angle). To solve for momentum we use vector components and add the X components and then can add the $Y$ components. This works for momentum because we established that momentum is conserved.
- Let's look at a diagram of what this looks like.
- Diagram -

- Ex. - A 2.00 kg ball, $A$, is moving at a velocity of $+5.00 \frac{\mathrm{~m}}{\mathrm{~s}}$. It collides with a stationary ball, $B$, also of mass 2.00 kg . After the collision ball A moves off at $30.0^{\circ}$ to the left of its original direction. Ball B moves off at $90.0^{\circ}$ to the final position of ball A. Draw a vector diagram to find the momentum of the balls after the collision and use this to solve for the velocity of each ball after the collision.

